

Subject: Microeconomic Theory I

Instructor Name: Muhammad Nouman Shafiq

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The Cost Function:

The cost function expresses a functional relationship between total cost and factors that determine it. Usually, the factors that determine the total cost of production (C) of a firm are the output (Q), the level of technology (T), the prices of factors (P_f) and the fixed factors (F). Symbolically, the cost function becomes

$$C=f(Q, T, P_f, F)$$

Such a comprehensive cost function requires multi-dimensional diagrams which are difficult to draw. In order to simplify the cost analysis, certain assumptions are made. It is assumed that a firm produces a single homogeneous good (q) with the help of certain factors of production. Some of these factors are employed in fixed quantities whatever the level of output of the firm in the short run. So they are assumed to be given.

The remaining factors are variable whose supply is assumed to be known and available at fixed market prices. Further, the technology which is used for the production of the good is assumed to be known and fixed. Lastly, it is assumed that the firm adjusts the employment of variable factors in such a manner that a given output Q of the good q is obtained at the minimum total cost, C.

Thus the total cost function is expressed as:

$$C=f(Q)$$

Which means that the total cost (C) is a function of output (Q), assuming all other factors as constant. The cost function is shown diagrammatically by a total cost (TC) curve. The TC curve is drawn by taking output on the horizontal axis and total cost on the vertical axis, as shown in Figure 1.

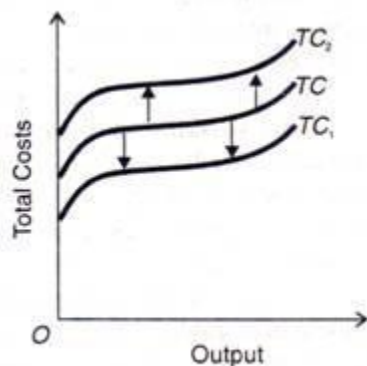


Fig. 1

It is a continuous curve whose shape shows that with increasing output total cost also increases. The total cost function and the TC curve relate total cost to output under given conditions. But if any of the given conditions such as the technique of production change, the cost function is changed.

For instance, if there is an improved technique of production, the cost of production for any given output will be less than before which will shift the new cost curve TC_1 below the old curve TC , as shown in Figure 1. On the other hand, if the prices of factors rise, the cost of production will increase which will shift the cost curve upwards from TC to TC_2 as shown in Figure 1.

Cost-Output Relation:

The Cost-output relation is discussed in the traditional and modern theories of costs under the short-run and long-run cost analysis which are explained as under.

The Traditional Theory of Costs:

The traditional theory of costs analyses the behaviour of cost curves in the short run and the long run and arrives at the conclusion that both the short run and the long run curves are U-shaped but the long-run cost curves are flatter than the short-run cost curves.

(A) Firm's Short-Run Cost Curves:

The short run is a period in which the firm cannot change its plant, equipment and the scale of organisation. To meet the increased demand, it can raise output by hiring more labour and raw materials or asking the existing labour force to work overtime.

Short-Run Total Costs:

The scale of organisation being fixed, the short-run total costs are divided into total fixed costs and total variable costs:

$$TC = TFC + TVC$$

Total Costs or TC:

Total costs are the total expenses incurred by a firm in producing a given quantity of a commodity. They include payments for rent, interest, wages, taxes and expenses on raw materials, electricity, water, advertising, etc.

Total Fixed Costs or TFC:

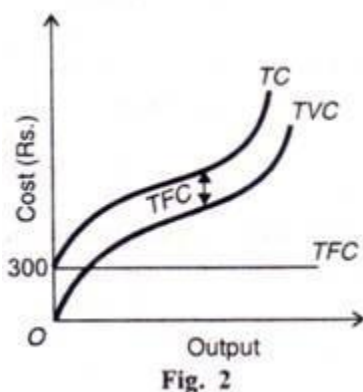
Are those costs of production that do not change with output. They are independent of the level of output. In fact, they have to be incurred even when the firm stops production temporarily. They include payments for renting land and buildings, interest on borrowed money, insurance charges, property tax, depreciation, maintenance expenditures, wages and salaries of the permanent staff, etc. They are also called overhead costs.

Total Variable Costs or TVC:

Are those costs of production that change directly with output. They rise when output increases, and fall when output declines. They include expenses on raw materials, power, water, taxes, hiring of labour, advertising etc., They are also known as direct costs.

The relation between total costs, variable costs and fixed costs is presented in Table 1, where column (1) indicates different levels of output from 0 to 10 units. Column (2) indicates that total fixed costs remain at Rs. 300 at all levels of output. Column (3) shows total variable costs which are zero when output is nothing and they continue to increase with the rise in output.

In the beginning they rise quickly, and then they slow down as the firm enjoys economies of large scale production with further increases in output and later on due to diseconomies of production, the variable costs start rising rapidly. Column (4) relates to total costs which are the sum of columns (2), and (3) i.e., $TC = TFC + TVC$. Total costs vary with total variable costs when the firm starts production.



The curves relating to these three total costs are shown diagrammatically in Figure 2. The TC curve is a continuous curve which shows that with increasing output total costs also increase. This curve cuts the vertical axis at a point above the origin and rises continuously from left to right. This is because even when no output is produced, the firm has to incur fixed costs.

TABLE 1 : COST FUNCTION IN THE SHORT-RUN

<i>TO</i>	<i>TFC</i>	<i>TVC</i>	<i>TC</i>	<i>AFC</i>	<i>AVC</i>	<i>ATC</i>	<i>MC</i>
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			(2+3)	(2÷1)	(3+1)	(5+6) or(4+1)	(from 4)
	Rs	Rs	Rs	Rs	Rs	Rs	Rs
0	300	0	300	300	0	300	—
1	300	300	600	300	300	600	300
2	300	400	700	150	200	350	100
3	300	450	750	100	150	250	50
4	300	500	800	75	125	200	50
5	300	600	900	60	120	180	100
6	300	720	1020	50	120	170	120
7	300	890	1190	42.9	127.1	170	170
8	300	1100	1400	37.5	137.5	175	210
9	300	1350	1650	33.3	150	183.3	470
10	300	2000	2300	30	200	230	650

The TFC curve is shown as parallel to the output axis because total fixed costs are the same (Rs. 300) whatever the level of output. The TVC curve has an inverted-S shape and starts from the origin O because when output is zero, the TVCs are also zero. They increase as output increases.

So long as the firm is using less variable factors in proportion to the fixed factors, the total variable costs rise at a diminishing rate. But after a point, with the use of more variable factors in proportion to the fixed factors, they rise steeply because of the application of the law of variable proportions. Since the TFC curve is a horizontal straight line, the TC curve follows the TVC curve at an equal vertical distance.

Short-Run Average Costs:

In the short run analysis of the firm, average costs are more important than total costs. The units of output that a firm produces do not cost the same amount to the firm. But they must be sold at the same price. Therefore, the firm must know the per unit cost or the average cost. The short-run average costs of a firm are the average fixed costs, the average variable costs, and the average total costs.

Average Fixed Costs or AFC equal total fixed costs at each level of output divided by the number of units produced:

$$AFC = TFC / Q$$

The average fixed costs diminish continuously as output increases. This is natural because when constant total fixed costs are divided by a continuously increasing unit of output, the result is continuously diminishing average fixed costs. Thus the AFC curve is a downward sloping curve

which approaches the quantity axis without touching it, as shown in Figure 3. It is a rectangular hyperbola.

Short-Run Average Variable Costs (or SAVC) equal total variable costs at each level of output divided by the number of units produced:

$$SAVC = TVC/Q$$

The average variable costs first decline with the rise in output as larger quantities of variable factors is applied to fixed plant and equipment. But eventually they begin to rise due to the law of diminishing returns. Thus the SAVC curve is U-shaped, as shown in Figure 3.

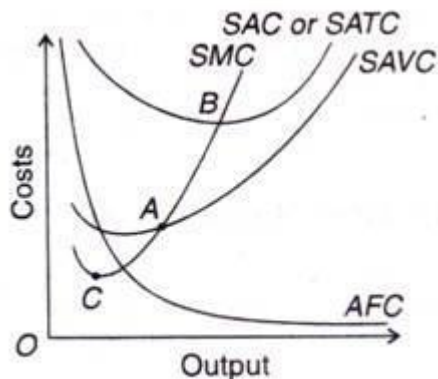


Fig. 3

Short-Run Average Total Costs (or SATC or SAC) are the average costs of producing any given output.

They are arrived at by dividing the total costs at each level of output by the number of units produced:

$$SAC \text{ or } SATC = TC/Q = TFC/Q + TVC/Q = AFC + AVC$$

Average total costs reflect the influence of both the average fixed costs and average variable costs. At first average total costs are high at low levels of output because both average fixed costs and average variable costs are large. But as output increases, the average total costs fall sharply because of the steady decline of both average fixed costs and average variable costs till they reach the minimum point.

This results from the internal economies, from better utilisation of existing plant, labour, etc. The minimum point B in the figure represents optimal capacity. As production is increased after this point, the average total costs rise quickly because the fall in average fixed costs is negligible in relation to the rising average variable costs.

The rising portion of the SAC curve results from producing above capacity and the appearance of internal diseconomies of management, labour, etc. Thus the SAC curve is U- shaped, as shown in Figure 3.

Why is SAC curve U-shaped?

The U-shape of the SAC curve can also be explained in terms of the law of variable proportions. This law tells that when the quantity of one variable factor is changed while keeping the quantities of other factors fixed, the total output increases but after some time it starts declining.

Machines, equipment and scale of production are the fixed factors of a firm that do not change in the short run. 'On the other hand, factors like labour and raw materials are variable. When increasing quantities of variable factors are applied on the fixed factors, the law of variable proportions operates.

When, say the quantities of a variable factor like labour are increased in equal quantities, production rises till fixed factors like machines, equipment, etc. are used to their maximum capacity. In this stage, the average costs of the firm continue to fall as output increases because it operates under increasing returns.

Due to the operation of the law of increasing returns when the variable factors are increased further, the firm is able to work the machines to their optimum capacity. It produces the optimum output and its average costs of production will be the minimum which is revealed by the minimum point of the SAC curve, point B in Figure 3.

If the firm tries to raise output after this point by increasing the quantities of the variable factors, the fixed factors like machines would be worked beyond their capacity. This would lead to diminishing returns. The average costs will start rising rapidly. Hence, due to the working of the law of variable proportions the short-run AC curve is U-shaped.

Short Run Marginal Cost:

A fundamental concept for the determination of the exact level of output of a firm is the marginal cost.

Marginal cost is the addition to total cost by producing an additional unit of output:

$$SMC = \Delta TC / \Delta Q$$

Algebraically, it is the total cost of $n + 1$ units minus the total cost of n units of output $MC_n = TC_{n+1} - TC_n$. Since total fixed costs do not change with output, therefore, marginal fixed cost is zero. So marginal cost can be calculated either from total variable costs or total costs. The result would be the same in both the cases. As total variable costs or total costs first fall and then rise, marginal cost also behaves in the same way. The SMC curve is also U-shaped, as shown in Figure 3.

Conclusion:

Thus the short-run cost curves of a firm are the SAVC curve, the AFC curve, the SAC curve and the SMC curve. Out of these four curves, the AFC curve is insignificant for the determination of the firm's exact output and is, therefore, generally neglected.

(B) Firm's Long-Run Cost Curves:

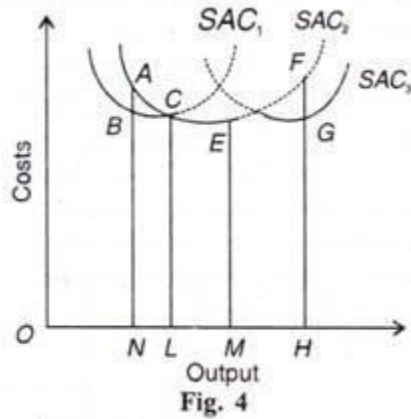
In the long run, there are no fixed factors of production and hence no fixed costs. The firm can change its size or scale of plant and employ more or less inputs. Thus in the long run all factors are variable and hence all costs are variable.

The long run average total cost or LAC curve of the firm shows the minimum average cost of producing various levels of output from all-possible short-run average cost curves (SAC). Thus the LAC curve is derived from the SAC curves. The LAC curve can be viewed as a series of alternative short-run situations into any one of which the firm can move.

Each SAC curve represents a plant of a particular size which is suitable for a particular range of output. The firm will, therefore, make use of the various plants up to that level where the short-run average costs fall with increase in output. It will not produce beyond the minimum short-run average cost of producing various outputs from all the plants used together.

Let there be three plants represented by their short-run average cost curves SAC_1 , SAC_2 and SAC_3 in Figure 4. Each curve represents the scale of the firm. SAC_1 depicts a lower scale while the

movement from SAC_2 to SAC_1 shows the firm to be of a larger size. Given this scale of the firm, it will produce up to the least cost per unit of output. For producing ON output, the firm can use SAC_1 or SAC_2 plant.



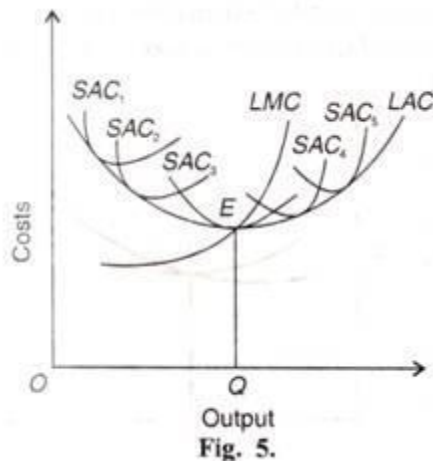
The firm will, however, use the scale of plant represented by SAC_3 since the average cost of producing ON output is NB which is less than NA , the cost of producing this output on the SAC_2 plant. If the firm is to produce OL output, it can produce at either of the two plants. But it would be advantageous for the firm to use the plant SAC_2 for the OL level of output.

But it would be more profitable for the firm to produce the larger output OM at the lowest average cost ME from this plant. However, for output OH , the firm would use the SAC_1 plant where the average cost HG is lower than HF of the SAC_2 plant. Thus in the long-run in order to produce any level of output the firm will use that plant which has the minimum unit cost.

If the firm expands its scale by the three stages represented by SAC_1 , SAC_2 and SAC_3 curves, the thick wave-like portions of these curves form the long-run average cost curve. The dotted portions of these SAC curves are of no consideration during the long run because the firm would change the scale of plant rather than operate on them.

But the long-run average cost curve LAC is usually shown as a smooth curve fitted to the SAC curves so that it is tangent to each of them at some point, as shown in Figure 5, where SAC_1 , SAC_2 , SAC_3 , SAC_4 and SAC_5 are the short-run cost curves. It is tangent to all the SAC curves but only to one at its minimum point.

The LAC is tangent to the lowest point E of the curve SAC_3 in Figure 5 at OQ optimum output. The plant SAC_3 which produces this OQ optimum output at the minimum cost QE is the optimum plant, and the firm producing this optimum output at the minimum cost with this optimum plant is the optimum firm. If the firm produces less than the optimum output OQ, it is not working its plant to full capacity and if it produces beyond it is overworking its plants. In both the cases, the plants SAC_2 and SAC_4 have higher average costs of production than the plant SAC_3



The LAC curve is known as the “envelope” curve because it envelopes all the SAC curves. According to Prof. Chamberlin, “It is composed of plant curves; it is the plant curve. But it is better to call it a “planning” curve because the firm plans to expand its scale of production over the long run.”

The long-run marginal cost (LMC) curve of the firm intersects SAC_1 and LAC curves at the minimum point E.

LAC Curve Flatter than SAC Curve:

Though the long-run average cost (LAC) curve is U-shaped, yet it is flatter than the short-run average cost (SAC) curve. It means that the LAC curve first falls slowly and then rises gradually after a minimum point is reached.

1. Initially, the LAC gradually slopes downwards due to the availability of certain economies of scale like the economical use of indivisible factors, increased specialisation and the use of

technologically more efficient machines or factors. The returns to scale increase because of the indivisibility of factors of production.

When a business unit expands, the returns to scale increase because the indivisible factors are employed to their maximum capacity. Further, as the firm expands, it enjoys internal economies of production. It may be able to install better machines, sell its products more easily, borrow money cheaply, procure the services of more efficient manager and workers, etc. All these economies help in increasing the returns to scale more than proportionately.

2. After the minimum point of the long-run average cost is reached, the LAC curve may flatten out over a certain range of output with the expansion of the scale of production. In such a situation, the economies and diseconomies balance each other and the LAC curve has a disc base.

3. With further expansion of scale, the diseconomies like the difficulties of coordination, management, labour and transport arise which more than counterbalance the economies so that the LAC curve begins to rise. This happens when the indivisible factors become inefficient and less productive due to the over expansion of the scale of production. Moreover, when supervision and coordination become difficult, the per unit cost increases. To these internal diseconomies are added external diseconomies of scale.

These arise from higher factor prices or from diminishing productivities of factors. As the industry continues to expand, the demand for skilled labour, land, capital, etc. rises. Transport and marketing difficulties also emerge. Prices of raw materials go up. All these factors lead to diminishing returns to scale and tend to raise costs.

Books

1. Pindyck and Rubinfeld with Mehta (2005), Microeconomics- latest available Edition in market.
2. D.N Dwivedi (2016), Microeconomics Theory and Application-- latest available Edition in market.
3. Koutsoyiannis, A., Modern Microeconomics, Macmillan, London.